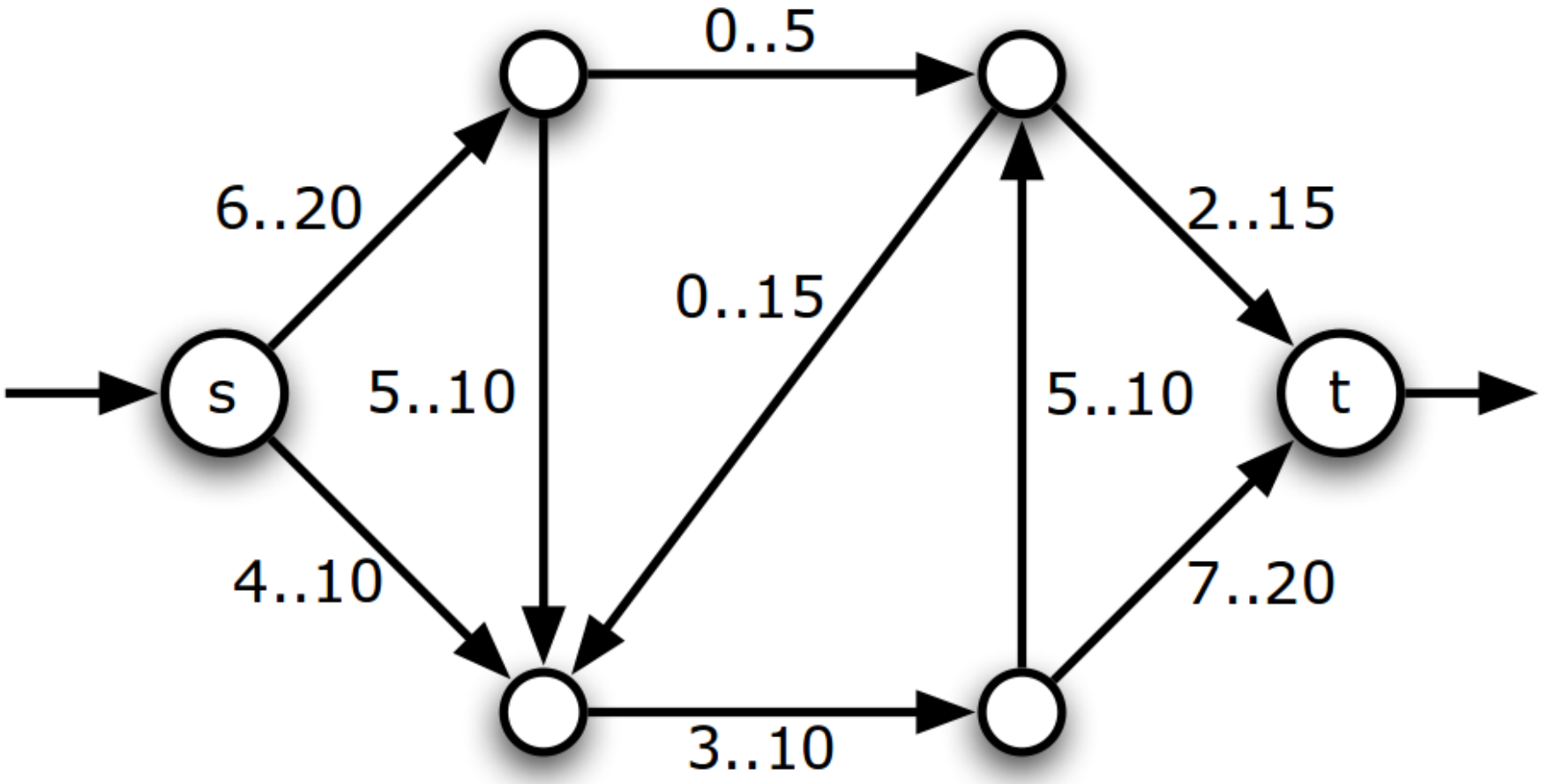
Tutorial 12
Advanced Graph Theory

November 13, 2013

1. NETWORK FLOWS

Determine if the following network has feasible flow.
Determine the flow function f for the network.



2. GRAPH COLOURING

2.1 Prove or disprove: Every k-chromatic graph has some proper k-coloring in which some color class has $\alpha(G)$ colors.

2.2 Prove $\chi(G) + \chi(G') \geq 2\sqrt{n}$

3. PLANARITY

3.1 Prove that a set of edges in a connected plane graph G forms a spanning tree of G if and only if the duals of the remaining edges form a spanning tree of G^*.

3.2 Prove that every n-vertex plane graph isomorphic to its dual has $2n - 2$ edges. For all $n \geq 4$, construct a simple n-vertex plane graph isomorphic to its dual.

3.3 Prove that every simple planar graph with atleast four vertices has at least four vertices with degree less than six.

4. HAMILTONIAN GRAPHS

4.1 Prove that the cartesian product of two Hamiltonian graphs in Hamiltonian. Conclude that the k-dimensional cube Q_k is Hamiltonian for $k \geq 2$.

5. NETWORK FLOWS

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